11.1A Notes Intro to Circles

**DEFINITION:** A **CIRCLE** is the set of all points that are equidistant from a fixed point, called the **CENTER**.

The **DISTANCE** between the **CENTER** and any **POINT** on the circle is called the radius.

**STANDARD FORM OF A CIRCLE:** $(x-h)^2+(y-k)^2=r^2$ Where $(h, k)$ is the center and $r$ is the radius.

**Example 1:**

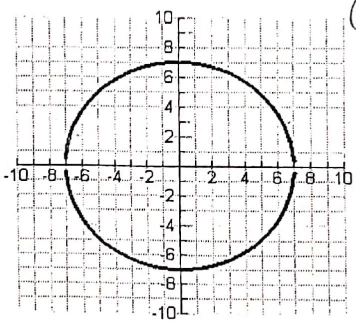
| | |
|---|---|
| a) Write the equation of a circle with a center (7, -3) and a radius of 6.<br>$(x-7)^2+(y+3)^2=36$ | b) Write the equation of a circle centered at the origin with radius of 11.<br>(0,0)<br>$x^2+y^2=121$ |

**Example 2:** Identify the center and radius of each circle.

| | |
|---|---|
| a) $(x-3)^2+y^2=\frac{25}{49}$<br>center ( 3 , 0 )<br>radius = $\frac{5}{7}$ | b) $(x-.5)^2+(y-.6)^2=81$<br>center ( 0.5, 0.6)<br>radius = 9 |

**Example 3: Given the graph, write the equation of the circle**

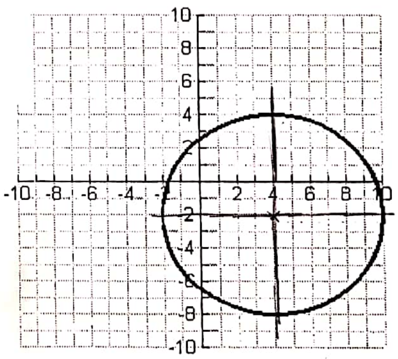
a)



center (0,0)
$r=7$

$x^2+y^2=49$

b)



center (4,-2)
$r=6$

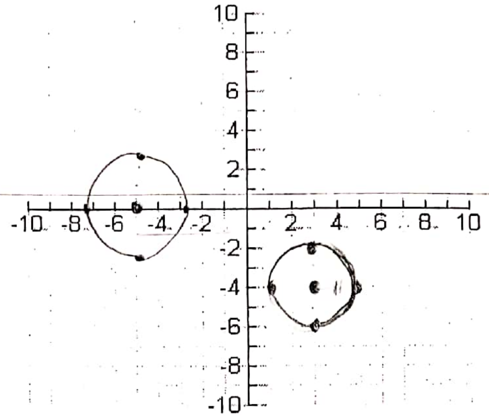
$(x-4)^2+(y+2)^2=36$

Example 4: Graph the circle.

a) $(x-3)^2 + (y+4)^2 = 4$

Radius = $2$

Center = $(3, -4)$




b) $(x+5)^2 + y^2 = 4.5$

Radius $\sqrt{4.5} = 2.12$

Center = $(-5, 0)$

Example 5: Write the equation for a circle with center at the origin & a point on the circle (1, 4).

$(0,0)$

**Sketch a picture:** **What do we need to find ?** radius

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(1-0)^2 + (4-0)^2 = r^2$$

$$(1)^2 + (4)^2 = r^2$$

$$1 + 16 = r^2$$

$$17 = r^2$$

$$\boxed{x^2 + y^2 = 17}$$

**Example 6:** Write the equation for a circle with center (-2, 3) and a point on the circle (1, -1).

$$(x+2)^2 + (y-3)^2 = r^2$$

$$(1+2)^2 + (-1-3)^2 = r^2$$

$$3^2 + (-4)^2 = r^2$$

$$9 + 16 = r^2$$

$$25 = r^2$$

$$5 = r$$

$$\boxed{(x+2)^2 + (y-3)^2 = 25}$$

Circles Worksheet-Review Name_________________________ Hour______

Goal: $(x-h)^2+(y-k)^2=r^2$

Rewrite each equation as a standard form equation of a circle. Then state the center and the radius of the circle.

1) $x^2-4x+y^2+6y-1=0$ $\quad +1 \quad +1$

$x^2-4x \underline{+4} + y^2+6y \underline{+9} = 1 \underline{+4} \underline{+9}$

$(-2)^2 \qquad (3)^2$

$(x-2)^2+(y+3)^2=14$

center $(2,-3)$

radius $=\sqrt{14}$

2) $x^2+y^2-10x+14y+65=0$ $\quad -65 \quad -65$

$x^2-10x \underline{+25} + y^2+14y \underline{+49} = -65$ $\quad +25 \quad \underline{+49}$

$(-5)^2 \qquad (7)^2$

$(x-5)^2+(y+7)^2=9$

center $(5,-7)$

radius $= 3$

You do

3) $x^2+y^2+2x+4y-31=0$

4) $x^2+y^2-4x+6y+4=0$

5) $x^2+y^2+8x+20y+112=0$

Notes Section 6.4 Polar Coordinates

## Polar Coordinate System

A **polar coordinate system** is a plane with a point $O$, the **pole**, and a ray from $O$, the **polar axis**, as shown in Figure 6.35. Each point $P$ in the plane is assigned **polar coordinates** $(r, \theta)$ as follows: $r$ is the **directed distance** from $O$ to $P$, and $\theta$ is the **directed angle** whose initial side is the polar axis and whose terminal side is the ray $\overrightarrow{OP}$.

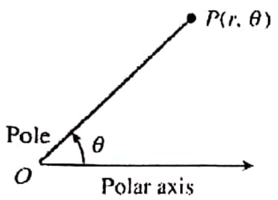



**FIGURE 6.35** The polar coordinate system.

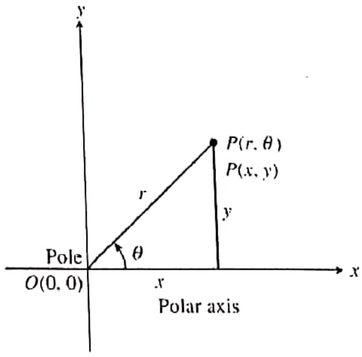



**FIGURE 6.38** Polar and rectangular coordinates for $P$.

IF $r > 0$ P is on terminal side of $\theta$
IF $r < 0$ p is on terminal side of $\theta + \pi$ (opp. dir.)

### Coordinate Conversion Equations

Let the point $P$ have polar coordinates $(r, \theta)$ and rectangular coordinates $(x, y)$. Then

$$x = r\cos\theta, \qquad r^2 = x^2 + y^2,$$

$$y = r\sin\theta, \qquad \tan\theta = \frac{y}{x}.$$

From Geometry

Equation of a circle
$(x-h)^2 + (y-k)^2 = r^2$
center of circle $(h, k)$
radius of circle $r$

From Algebra

· Complete the square

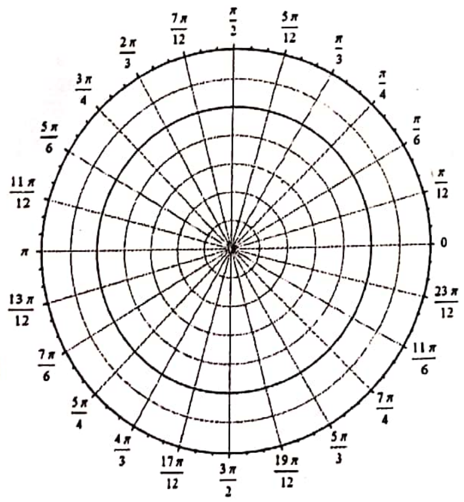
$x^2 + bx + \_\_\_\_ = \_\_\_\_$

$\left(\frac{b}{2}\right)^2$

$\left(x + \frac{b}{2}\right)^2$

perfect sq trinomial that factors!

Polar Coordinate Plane

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Polar $(r, \theta)$ rectangular $(x, y)$

Converting from Polar to Rectangular Coordinates ★ If angle $\theta$ is a unit circle angle give exact values!

Ex 1) Use an algebraic method to find the rectangular coordinates of the point with the given polar coordinates. Approximate the exact solution values with a calculator when appropriate (Round two decimal places)

u.c a) $\left(3, \frac{4\pi}{3}\right)$ ($r$, $\theta$)

$x = r\cos\theta$

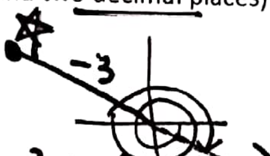
$y = r\sin\theta$

$\left(3\cos\frac{4\pi}{3}, 3\sin\frac{4\pi}{3}\right)$

$\left(3 \cdot -\frac{1}{2}, 3 \cdot -\frac{\sqrt{3}}{2}\right)$

$\left(-\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$

calc b) $(-3, -29\pi/7)$



$\left(-3\cos\left(-\frac{29\pi}{7}\right), -3\sin\left(-\frac{29\pi}{7}\right)\right)$

$(-2.70, 1.30)$

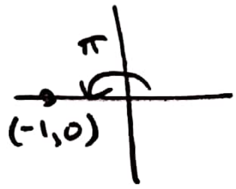
u.c c) $(-2, 120°)$

$(-2\cos 120°, -2\sin 120°)$

$\left(-2 \cdot -\frac{1}{2}, -2 \cdot \frac{\sqrt{3}}{2}\right)$

$\boxed{(1, -\sqrt{3})}$

u.c d) $(-2, \pi)$



$(-2\cos\pi, -2\sin\pi)$

$(-2 \cdot -1, -2 \cdot 0)$

$\boxed{(2, 0)}$

Ex 2) Rectangular coordinates of a point P are given. Use an algebraic method, and approximate exact solutions with a calculator when appropriate, to find all polar coordinates of P that satisfy the given condition.

$r^2 = x^2 + y^2$

$\tan\theta = \frac{y}{x}$

a) $[0, 2\pi]$

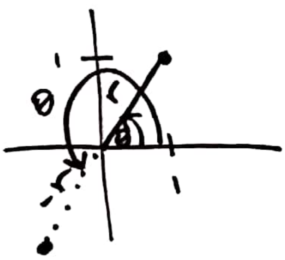
P = ( 1, 1 ) ($x$, $y$)

$r^2 = 1^2 + 1^2$

$\sqrt{r^2} = \sqrt{2}$

$r = \pm\sqrt{2}$

$\left(\sqrt{2}, \frac{\pi}{4}\right)\left(-\sqrt{2}, \frac{5\pi}{4}\right)$



$\theta = \tan^{-1}\left(\frac{1}{1}\right)$

$\theta = \frac{\pi}{4}$

$\theta = \frac{\pi}{4} + \pi$

b) $-\pi \leq \theta \leq \pi$

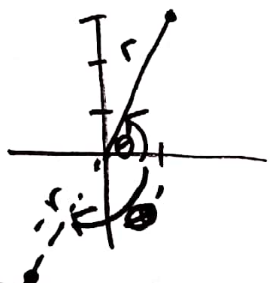
P = ( 1, 3 ) ($x$, $y$)

$r^2 = 1^2 + 3^2$

$= 1 + 9$

$\sqrt{r^2} = \pm\sqrt{10}$

$r = \pm\sqrt{10}$



$(\sqrt{10}, 1.25)(-\sqrt{10}, -1.89)$

$\theta = \tan^{-1}\left(\frac{3}{1}\right)$

$\theta = 1.25$ rad

$-\pi$